The pressure distribution analysis for short hydrodynamic journal bearing with the effect of centrifugal force of lubricant

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KEYWORDS

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Short-bearing theory
Hydrodynamic bearings

ABSTRACT

The paper aims to obtain the analytical pressure distribution of hydrodynamic journal bearings. In the classical short-bearing theory, the pressure distribution was studied by ignoring the effect of the centrifugal force of the lubricating fluid film. However, the self-oscillation of the shaft of high-power motors does not follow the rules in classical hydrodynamic lubrication theory. To explain this phenomenon, a new form of the Reynolds equation, in which the influence of the centrifugal force of the lubricant is not ignored, is established. The oil's pressure in the hydrodynamic journal bearing is obtained by solving the new Reynolds equation. The numerical results are considered in the case of the stable equilibrium position of the motion of the shaft, i.e. the symmetry axis of the shaft does not move. The plots of the pressure distribution in the tangential direction, axial direction and 3D are displayed too.

1.0 INTRODUCTION

One of the problems of lubrication theory is that the self-oscillation of the shaft, which is mainly caused by the nonlinear hydrodynamic force of the lubricating oil film. To investigate the dynamics as well as the stability of motion of the shaft, we must calculate these hydrodynamic forces. These forces are obtained by integral the pressure distribution function of the oil film over the surface of the rotating shaft. So the core of this problem is investigation of the oil's pressure in hydrodynamic journal bearing. In classical hydrodynamic lubrication theory (Loitsyanskii, 1987 and Hori, 2002), the Reynolds equation is established by using the system of hypothesis. In which the author pays attention to the hypothesis about ignoring the centrifugal force of lubricating oil film. However, the phenomenon of self-oscillation and the dynamic instability of
the shafts of high-power motors do not follow the rules in classical hydrodynamic lubrication theory. In industry, the angular velocity of the shaft is very large, so the influence of the centrifugal force of the lubricating oil film cannot be ignored. To explain this phenomenon, the author proposes a plan to establish a new form of Reynolds equation, in that the influence of centrifugal force of the lubricating oil film is not ignored (Belyaev, 2008 and Nguyen, 2011). The investigation of dynamics of the shaft can be launched by this new equation.

In classical hydrodynamic lubrication theory, the object of study is a shaft that rotates in a fixed bearing. The narrow gap between the shaft and the bearing is filled by a thin film of oil. However, when considering the dynamic of the shaft in hydrodynamic bearings, two types of bearings are classified: common hydrodynamic bearing (two -solid body model) (Tondl, 1971) and hydrodynamic bearing with floating bush (three-solid body model) (Hatakenaka, 2002 and Boyaci, 2008). To approach these two types of bearings simultaneously when researching the dynamics of the shaft, we consider the general model. The shaft is an absolute rigid cylinder, which rotates free with angular speed inside the bearing modeled as a cylindrical shell rotating in the same direction with angular speed. The rotating axis of the cylindrical shell is fixed. The narrow gap between two solid bodies is filled with the pre-stressed oil with constant dynamic viscosity. With this general model, the research as a lemma to solve the dynamic problem for hydrodynamic bearings with floating ring.

The pressure distribution of the lubricating oil film is one of the important problems of hydrodynamic lubrication theory. Therefore, this problem is studied very early with many different case of study. Sommerfeld was the first to obtain the pressure distribution after solving Reynolds equation under the assumption that there is no lubricant flow in the axial direction (assumption of an infinitely long bearing) (Sommerfeld, 1959). Sommerfeld's hypothesis and his solution are also applied (Korovtrinskii, 1969). Ocvirk solution for infinitely short bearing assumption neglects circumferential pressure gradients (first term of Reynolds equation) (Ocvirk, 1953). The effect of the change in bearing radial clearance with pressure variation of the fluid (lubricant) in a hydrodynamic journal bearing was carried out (Erhunmwun, 2019). This study was carried out using the Galerkin Finite Element Method (GFEM) for solution of the classical Reynolds Equation. The effects of oil supply pressure on the temperature and pressure at different groove locations on a hydrodynamic journal bearing were investigated (Mohamad, 2014). In this study, the experimental measurements of temperature and pressure profiles for different oil supply pressure values at different groove locations in the hydrodynamic journal bearing were carried out. Increasing the oil inlet supply pressure increased the lubricant pressure profiles in the journal bearing. Changing the oil groove locations from the initial position tended to reduce the pressure profiles. (Kittipong, 2020) studies experimentally the oil pressures on the pad of thrust bearings under hydrodynamic lubrication. The smooth pad and the pad with circumferential grooves surface were considered. In this study, the experimental results showed that the oil pressure distribution is affected by the axial load and angular velocity. The results of numerical solution Reynolds equation for laminar, steady oil flow in slide bearing with micropolar structure is presented (Pawel, 2011). The distribution of pressure on the journal bearing, considering the cavitation and non-cavitation model applied for two different forms of lubricant which are engine oil (SAE20W40) and palm oil (Rasep et al., 2021). In this study, at a speed of 1500 rpm, results are obtained by CFD method with an eccentricity ratio equal to 0.8 and L/D ratio equal to 1.0. The mechanical and thermal deformations effects were investigated by numerical method in the works (Li et al., 2018 and Tauviqirrahman et al., 2021).
2.0 THE LUBRICATION THEORY WITH THE EFFECT OF CENTRIFUGAL FORCE OF LUBRICANT

2.1 Governing Equations

To approach the floating ring bearings, we consider the general model, Figure 1.

The shaft is an absolute rigid cylinder \((O_1, R_1)\), which rotates free with angular speed \(\omega_1\) inside the bearing modeled as a cylindrical shell \((O_2, R_2)\) rotating in the same direction with angular speed \(\omega_2\). The rotating axis of the cylindrical shell is fixed. The narrow gap between two solid bodies is filled with a pre-stressed oil with constant dynamic viscosity.

Continuity equation for incompressible fluid \((\rho = \text{const})\):

\[
\nabla \cdot \mathbf{v} = 0. \tag{1}
\]

Integration of the local form of the continuity equation (1) for the incompressible fluid for the gap \(R_1 \leq r \leq R_1+h\) yields (Belyaev, 2008):

\[
\frac{1}{R_1+h} \frac{\partial}{\partial \phi} q_\phi + \frac{\partial}{\partial z} q_x + \frac{R_1}{R_1+h} \frac{\partial}{\partial t} h = 0, \tag{2}
\]

in which

\[
q_\phi = \int_{R_1}^{R_1+h(\phi)} v(r) dr ; \quad q_x = \int_{R_1}^{R_1+h(\phi)} \frac{r}{R_1+h} w(r) dr. \tag{3}
\]

\(u, v, w\) – are the velocity components of the fluid flow.
Integration of the Navier–Stoke equation obtains the velocity components of the fluid flow. Replace their velocity components to continuity Equation (2), (3) yields the Reynolds equation examining the effect of the centrifugal force of lubricating oil film:

\[
\frac{h^3}{6} \frac{\partial^2 P}{\partial z^2} + \frac{h^3}{6} \frac{1}{R_i + h} \left[ \frac{\partial^2 P}{\partial \varphi^2} + \frac{3}{h} \frac{\partial P}{\partial \varphi} \frac{\partial h}{\partial \varphi} \right] = \mu \frac{R_i}{R_i + h} \left[ \frac{2}{\partial t} + (\omega_i + \omega_2) \frac{\partial h}{\partial \varphi} \right],
\]

in which

\[
p = \int \rho \frac{v^2}{r} dr + P(\varphi, z).
\] (5)

In this equation (5), \(p\) – pressure distribution of hydrodynamic journal bearings with the effect of the centrifugal force of lubricating oil film. \(P(\varphi, z)\) is the axial and tangential pressure distribution component. In the classical lubrication theory, the pressure of oil film is assumed to be constant in the radial direction, i.e. \(p = P(\varphi, z)\).

### 2.2 Short Bearing Type Approach

To solve equation (4), consider the case of a short bearing with a narrow clearance, i.e.:

\[
\frac{L}{D} \ll 1 \Rightarrow \left( \frac{L}{D} \right)^2 \rightarrow 0.
\] (6)

in which \(L\) and \(D\) are the length and diameter of the bearing, respectively. Under the assumption that the change in pressure in the circumferential direction (\(\varphi\)) is much smaller than in the axial direction (\(z\)) and neglecting the term \(h/R_i\). The Reynolds equation to the form:

\[
\frac{h^3}{6} \frac{\partial^2 P}{\partial z^2} = \mu \left[ \frac{2}{\partial t} + (\omega_i + \omega_2) \frac{\partial h}{\partial \varphi} \right].
\] (7)

This is a quadratic differential equation that can be solved by analytical method. The two integration constants in Equation (7) can be determined by the boundary condition and the symmetry condition:

\[
\begin{align*}
z &= 0, \ r = R_2: & p &= p^a; \quad \text{let } \varphi = \frac{L}{2}, \quad p = 0, \quad \text{let } z = 0. \\
p(z) &= p(-z), \forall {\varphi} \in \left[ 0, \frac{L}{2} \right].
\end{align*}
\] (8)

Integration of Equation (7) yields:

\[
P(\varphi, z) = \frac{6\mu}{h^3} \left[ \frac{2}{\partial t} + (\omega_i + \omega_2) \frac{\partial h}{\partial \varphi} \right] \left( \frac{1}{2} z^2 + C \right),
\] (9)
in which the local oil film thickness:

\[ h = h_0 - e \cos \varphi; \quad h_0 = R_2 - R_1. \]  

(10)

The pressure in the gap is given by Equation (5) and it allows one to determine the integration constant \( C \) in Equation (9) and obtain the pressure distribution in the whole of the gap:

\[
p(r, \varphi, z, t) = p + \frac{3\mu}{h^3} \left[ (\omega + \omega_z - 2\dot{\gamma})e \sin(\varphi - \gamma) - 2e \cos(\varphi - \gamma) \right] \left( z^2 - \frac{L^2}{4} \right).
\]

(11)

According to the point of view of the classical theory of a short bearing, the pressure disappears at both ends of the bearing. That is, it is not the absolute pressure that is of interest, but the excess pressure distribution:

\[
\bar{p} = p - p = \frac{3\mu}{h^3} \left[ (\omega + \omega_z - 2\dot{\gamma})e \sin(\varphi - \gamma) - 2e \cos(\varphi - \gamma) \right] \left( z^2 - \frac{L^2}{4} \right).
\]

(12)

In Equation (12), the excess pressure distribution does not depend on the radius, and moreover, it does not yet depend on the given pressure on the inner surface of the rotating cylinder \( p_{\text{in}} \) and the density of the oil \( \rho \).

3.0 NUMERICAL RESULTS AND DISCUSSION

The parameters used in this analysis are as shown in Table 1:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Short bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lengths of bearing, ( L ) (m)</td>
<td>45 \times 10^{-3}</td>
</tr>
<tr>
<td>Diameter of journal, ( D ) (m)</td>
<td>100 \times 10^{-3}</td>
</tr>
<tr>
<td>Radial clearance, ( h_0 ) (m)</td>
<td>0.1 \times 10^{-3}</td>
</tr>
<tr>
<td>Eccentricity, ( e ) (m)</td>
<td>0.05 \times 10^{-3}</td>
</tr>
<tr>
<td>Eccentricity ratio, ( \varepsilon )</td>
<td>0.5</td>
</tr>
<tr>
<td>Speed of journal, ( \omega_1 ) (rpm)</td>
<td>300÷2000</td>
</tr>
<tr>
<td>Dynamic viscosity of oil, ( \mu ) (Pa.s)</td>
<td>0.19</td>
</tr>
<tr>
<td>Inlet temperature (°C)</td>
<td>25</td>
</tr>
</tbody>
</table>
For simplicity, we consider the case of: the equilibrium position of the motion of the shaft is stable, i.e. the axis of the shaft does not move. Then, the eccentricity $O_1O_2 = e(t)$ and the angle of rotation $\gamma(t)$ will be constants, Figure 1.

\[
\begin{align*}
& e = e^* = \text{const}; \\
& \gamma = \gamma^* = \text{const},
\end{align*}
\]

(13)

For a numerical calculation, we take $e^* = 0.05 \times 10^{-3} \text{m}$; $\gamma^* = 3\pi/4$. It is easy to see that eccentricity ratio $\varepsilon = 0.5$. Under the substitution of the relation (13) in to Equation (12) obtain the pressure distribution:

\[
\bar{p} = \frac{3\mu}{h^3} \left( \omega_1 + \omega_2 \right) e^* \sin \theta \left( z^2 - \frac{L^2}{4} \right),
\]

(14)

in which $\theta = \varphi - \gamma^*$, Figure 1.

Pressure distribution in the clearance at the cross-section in the middle of the bearing

Figure 2: The schema of the bearing: the points A, B, C, D, E, respectively angle $\theta = 0, \pi/5, \pi/2, \pi, 3\pi/2$

Formula (14) follows the function of pressure on these sections:

\[
\bar{p} \bigg|_{z=0} = \frac{3\mu L^2}{4h^3} \left( \omega_1 + \omega_2 \right) e^* \sin \theta.
\]

(15)

\[
\bar{p} \bigg|_{z=L/4} = -\frac{9\mu L^2}{16h^3} \left( \omega_1 + \omega_2 \right) e^* \sin \theta.
\]

(16)

The graphs of these functions are shown in Figure 3.
Based on Figure 3, the pressure distribution obtained following conclusions:

(a) The excess pressure is positive in half of the gap $\theta \in [\pi, 2\pi]$, and in the other half of the gap $\theta \in [0, \pi]$, the excess pressure turns out to be negative. The excess pressure disappears at both points A and D, Figure 2. The result from the analytical solution was validated by comparing it with other results from FEM (Erhunmwun, 2019) and CFD (Rasep et al. 2021). The comparison shows a strong positive correlation. In work (Pawel, 2011), the dimensionless pressure distributions $p$ in dependence on coupling number $N_2$ by micropolar ($N_2>0$) and Newtonian ($N_2=0$) lubrication are displayed similarly.

(b) When the rotor rotates quickly, i.e., $\omega_1$ large and the radial clearance $h_0$ is narrow enough, the maximum pressure in the gap reaches at point B, is large enough (Hori, 2002).

(c) By increasing $z$, the maximum pressure in the gap decreases. That is, the maximum pressure is reached at the center of the bearing and gradually decreases towards its ends. The excess pressure disappears at both ends of the bearing $z=\pm L/2$. The pressure distribution in the clearance along the bearing axis is obtained.
Based on Figure 5, the pressure distribution obtained following conclusions:

(a) It is easy to see that on lines $a$ and $d$, Figure 4, the excess pressure is equal to zero, that is, on these lines the pressure in the gap is constant and equal to the pressure at the ends of the bearing.

(b) Figure 5a and Figure 5b show the excess pressure on lines $b$ and $c$, Figure 4, which are zero at the ends of bearing. It has reached a minimum in the middle of the bearing. This analysis shows that, there is an oil flow from the ends of bearing to its middle. This lubricant flow along the rotating axis of the shaft and it is the Poiseuille flow.

(c) On the opposite, Figure 5c shows excess pressure on lines $e$, Figure 4, is equal to zero at the ends of bearing and has reached the maximum in the middle of the bearing. Similarly, there is a flow of oil from the middle of the bearing to its ends. In three-dimensional plot Figure 5d, the excess pressure in the whole of the gap in the general case of formula (16).
Figure 5: The plot of the pressure distribution along the axis of the bearing: (a) on the line $b$, (b) on the line $c$, (c) on the line $e$, (d) in the whole of clearance of bearing, Figure 4.
CONCLUSIONS
This paper obtains the expression of the pressure distribution in the hydrodynamic journal bearings by analytical method. The pressure distribution is obtained by solving the new Reynolds equation in the case of short bearing type approach. In this equation, the effect of centrifugal force of the lubricating oil film is mentioned. The plot of the pressure distribution in the gap is completely consistent with the boundary conditions and physical properties of the flow. The pressure distributions in the clearance at the cross-section and along the bearing axis are obtained. In three-dimensional plot, the excess pressure in the gap versus circumferential coordinate (θ) and the z-direction (z) is displayed, too. The result from the analytical solution was validated by comparing it with other result from FEM and experimental method. The comparison shows a strong positive correlation.

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