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A study of nonlinear transient behavior of worn out 3-lobe nonrecessed journal bearing

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KEYWORDS

Nonlinear
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Bearing
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ABSTRACT

The purpose of this work is to discuss how the nonlinear transient behavior of a symmetric 3- lobe non-recessed hybrid journal bearing is affected by the worn bearing surface. The stability of a non-recessed worn out 3-lobe journal bearing has been investigated using the RungeKutta technique to solve nonlinear equations of motion. For various wear depth parameter values, the journal center trajectories have been plotted. According to the stability study's findings, a 3-lobe hybrid journal bearing's stability is severely influenced by the wear defect. The selection of the bearing's non-circular shape and worn-out surface parameter must be correct in order to sustain stability under dynamic operating conditions.

1.0 INTRODUCTION

The stability analysis of journal bearings is crucial to ensure reliable and efficient operation in various rotary machines, such as turbines and engines. Stability refers to the ability of the bearing to maintain a consistent and predictable position during operation, avoiding undesirable vibrations and whirl. The non-circular/lobe journal bearings are preferred over circular bearings due to better stability. The literature on non-circular/lobe journal bearings has seen significant growth in recent years, reflecting the increasing interest in their potential advantages and diverse applications (Lund and Thomsen, 1978; Goyanka and Booker, 1983; Ghosh et al., 2003; Phalle et al., 2011; Zare, 2021; Kushare and Sharma, 2013; Singh and Awasthi , 2022). Non-circular/ Lobed hydrostatic/ hybrid journal bearings remain widely used and effective in many applications, especially when properly designed and operated within their stable operating range (Ghosh et al.,

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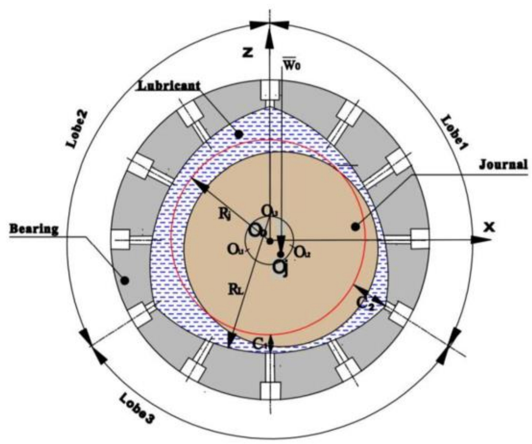
2003; Goyanka and Booker,1883; Koshare and Sharma, 2013). As researchers continue to explore and optimize these unconventional bearing geometries, further advancements are likely to occur, contributing to the development of more efficient, stable, and reliable bearing solutions.

During continuous starts and stops, circular, 2-lobe and 3-lobe journal bearings are subjected to repeated cycles of acceleration, deceleration, and changes in operating conditions. This dynamic and transient operation can significantly impact the wear characteristics of the journal bearing surfaces. The rapid changes in operating conditions during starts and stops can lead to worn out bearing surfaces (Dufrane et al., 1983; Hashimoto et al., 1986). This can result in insufficient lubricant film thickness and increased metal-to-metal contact, causing abrasive wear. Thus, consideration of the influence of wear is of utmost importance in the analysis of non-circular journal bearings. The significant research in both experimental and analytical domains in the field of circular and 2-lobe worn out journal bearing were available and documented in written works of literature (Dufrane et al., 1983; Hashimoto et al.,1986;Fillon and Bouyer,2004; Awasthi et al., 2007; Phalle et al.,2011; Singh and Sharma,2021).

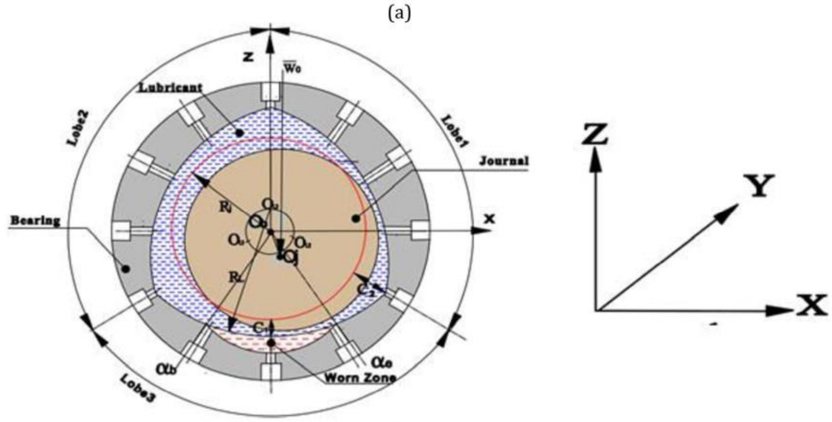
Dufrane et al., (1983) developed the first analytical model, later on it was experimentally validated for the worn-out hydrodynamic journal bearing surface. They asserted that symmetric wear damage occurs at the bottommost surface of the journal bearing. Referring to Dufrane's model, many researchers (Hashimoto et al., 1986; Fillon and Bouyer, 2004; Awasthi et al., 2007; Phalle et al., 2011) have studied the performance of worn journal bearing. Singh and Sharma, (2021) have investigated various aspects of wear-induced effects on journal bearing behavior focusing on both experimental and analytical investigation. Redcliff and Vohr, (1969) explored the occurrence of wear in externally pressurized journal bearings and investigated the wear mechanisms, effects on performance, and potential mitigation strategies. Phalle et al., (2011) investigated the concern of damaged bearing surface on the performance of 2-lobe recessed journal bearing also presented the performance characteristics for different restrictors.

The focus on transient analysis of hydrostatic and hybrid journal bearing in the published literature (Pang et al.,1993; San Andres, 1997; Kushare and Sharma, 2014; Sharma and Kushare,2017) has been relatively limited compared to hydrodynamic journal bearing (Newkirk and Talor,1925; Yadhav and Ram,2018; Sinhasan and Goyal,1992; Raghunandana and Majumdar,1999;Choy et al.,1991; Huang et al.,2017). Pang et al., (1993) carried out investigations to understand how the hydrostatic bearing adapts to sudden load changes and how the compressibility of the oil impacts the transitory movement of journal. San Andres, (1997) investigated the transitory behaviour in hydrostatic mode for pocket shaped bearing when subjected to various external loads. Recently, Kushare and Sharma, (2014) and Sharma and Kushare,(2017) performed the non-linear transitory journal motion study for 2-lobe worn journal bearing oiled with variable viscosity lubricant and taking the bearing roughness into consideration, respectively. They found that the stability parameters derived from linear equation of motion trajectory study do not provide precise margin compared to nonlinear stability study.

A review of existing literature indicates that nonlinear analysis predicts accurate stability margins compared to linear analysis. Thus, in order to accurately estimate journal motion response; in this study nonlinear model have been used. Research in this area is relatively limited and there hasn't been any research on non-linear transient stability, yet which focuses on transient stability behavior of worn out 3-lobe non-recessed bearing system. Therefore, the current study aims to investigate how wear, which occurs over time, affects the stability of 3-lobe non-recessed journal bearing system and provide the insights about for accurate and reliable predictions, and informed decision-making during the design and operation of 3-lobe non-recessed hybrid journal bearing systems. The 3-lobe bearing geometry used in the analysis study is shown in Figure.1 (a) & (b).



(a)



(b)

Figure 1: (a) The 3-lobe bearing geometry used in the analysis study (b) the worn region of the bearing.

2.0 ANALYSIS

2.1 Lubricant Fluid Field Reynolds Equation:

The lubricant flow field for 3-lobe journal bearing governed by the modified Reynolds equation is stated in non-dimensional form (Dawson, 1962; Chaudhary et al., 2021; Phalle et al., 2011; Garg, 2015).

$$\frac{\partial}{\partial \alpha}\left(\bar{h}^3 \bar{F}_2 \frac{\partial \bar{p}}{\partial \alpha}\right)+\frac{\partial}{\partial \beta}\left(\bar{h}^3 \bar{F}_2 \frac{\partial \bar{p}}{\partial \beta}\right)=\Omega \frac{\partial}{\partial \alpha}\left(1-\frac{\bar{F}_1}{\bar{F}_0}\right)\bar{h}+\frac{\partial \bar{h}}{\partial \bar{t}} \tag{1}$$

where $\bar{F}_0$, $\bar{F}_1$ and $\bar{F}_2$ are given by the following relations:

$$\bar{F}_0 = \int_0^1 \frac{1}{\bar{\mu}} d\bar{z}, \quad \bar{F}_1 = \int_0^1 \frac{\bar{z}}{\bar{\mu}} d\bar{z}, \quad \bar{F}_2 = \int_0^1 \frac{\bar{z}}{\bar{\mu}}\left(\bar{z} - \frac{\bar{F}_1}{\bar{F}_0}\right) d\bar{z}, \tag{2}$$

2.2 Fluid Film Thickness Equation for Worn Journal Bearings:

The schematic representation in Figure.1 (b) illustrates the worn region of the bearing. Dufrane's abrasive wear model is used to describe the wear phenomenon. The visual observation and assumption made by Dufrane et al., (1983) state that the bearing's bottom exhibits a symmetrical wear pattern and a consistent wear pattern along its axial length. Thus, the film thickness for 3-lobe worn journal bearing is specified as (Phalle et al., 2011; Kushare and Sharma, 2013):

$$\bar{h} = \bar{h}_0 + \partial\bar{h} \tag{3}$$

Where, $\bar{h}_0 = 1/\delta - (\bar{X}_J - \bar{X}_L^i)\cos\propto - (\bar{Z}_J - \bar{Z}_L^i)\sin\propto$ (4)

and $\partial\bar{h} = \bar{\delta}_w - 1 - \sin\alpha$ for worn bush geometry and $\partial\bar{h} = 0$ for unworn geometry.

2.3 Governing Equations for Capillary Restrictor:

The non-dimensional expression for flow of lubricant through the capillary compensating device is given as (Phalle et al., 2011; Sharma and Kushare, 2017) :

$$\bar{Q}_R = C_{s2}(1 - \bar{p}_c) \tag{5}$$

2.4 Stability Parameters:

For 3-lobe bearing, journal dimensionless critical mass is illustrated as follows: (Sinhasan and Goyal, 1992; Jagadeesha et al., 2012; Thang, 2022)

$$\bar{M}_c = \frac{\bar{G}_1}{\bar{G}_2 - \bar{G}_3} \tag{6}$$

Where, $\bar{G}_1 = \left[\bar{C}_{xx}\bar{C}_{zz} - \bar{C}_{zx}\bar{C}_{xz}\right]$

$$\bar{G}_2 = \frac{\left[\bar{S}_{xx}\bar{S}_{zz} - \bar{S}_{zx}\bar{S}_{xz}\right]\left[\bar{C}_{xx} + \bar{C}_{zz}\right]}{\left[\bar{S}_{xx}\bar{C}_{zz} + \bar{S}_{zz}\bar{C}_{xx} - \bar{S}_{xz}\bar{C}_{zx} - \bar{S}_{zx}\bar{C}_{xz}\right]}$$

$$\bar{G}_3 = \frac{\left[\bar{S}_{xx}\bar{C}_{xx} + \bar{S}_{xz}\bar{C}_{xz} + \bar{S}_{zx}\bar{C}_{zx} + \bar{S}_{zz}\bar{C}_{zz}\right]}{\left[\bar{C}_{xx} + \bar{C}_{zz}\right]}$$

The journal speed at the instability threshold is expressed as (Sinhasan and Goyal, 1992; Jagadeesha et al., 2012)

$$\overline{\omega}_{th} = \left[\overline{M}_c \big/ \overline{F}_o \right]^{1/2} \tag{7}$$

To investigate the stability of the 3-lobe journal bearing, the transient response is analyzed using nonlinearized equations of motion. A nonlinear equation that expresses the disturbed motion of the journal in terms of the instantaneous fluid-film force components is stated as: (Zare et al., 2020; Sinhasan and Goyal, 1992; Kushare and Sharma, 2017):

$$\begin{bmatrix} \overline{M}_J & 0 \\ 0 & \overline{M}_J \end{bmatrix} \begin{Bmatrix} \ddot{\overline{X}}_J \\ \ddot{\overline{Z}}_J \end{Bmatrix}_{|\bar{t}} = \begin{Bmatrix} \Delta\overline{F}_x \\ \Delta\overline{F}_z \end{Bmatrix}_{|\bar{t}} = \begin{Bmatrix} \overline{F}_x - \overline{F}_{xo} \\ \overline{F}_z - \overline{F}_{zo} \end{Bmatrix}_{|\bar{t}} \tag{8}$$

Where, $\overline{F}_{xo}$ and $\overline{F}_{zo}$ are the equilibrium state fluid-film force components.

To obtain the nonlinear trajectory of the system, The fourth order Runge-Kutta technique is used when the journal position is first perturbed. This numerical method allows for the integration of the nonlinear equations of motion, providing valuable insights into the system's behavior during transient response analysis. The boundary conditions employed have already been discussed in the earlier published literature of the author (Kushare and Sharma, 2017).

3.0 RESULTS AND DISCUSSION

The computation was performed using the solution procedure presented in the overall self-explanatory iterative procedure presented in Figure 2. The selection of bearing geometry and operating parameters was carefully done based on published literature to ensure their appropriateness (Dufrane et al., 1983; Hashimoto et al., 1986; Fillon and Bouyer, 2004; Awasthi et al., 2007) as presented in Table.1

As mentioned, there are no published literature studies available for transient journal motion stability of 3-lobe symmetric non-recessed worn hybrid bearing. In order to assess the accuracy and reliability of the developed numerical model, The current study's simulated results are validated against previously published results from Hashimoto et al., (1986) and Sinhasan and Goyal, (1992). The numeric model developed for the worn-out bearing is confirmed by matching the published results of Hashimoto et al., (1986) for hydrodynamic journal bearing and are presented in Figure 3.

Table 1: The selection of bearing geometry and operating parameters.

Geometric Parameters	
Aspect Ratio (λ)	1.0
Land Width Ratio ($\overline{a}_b$)	0.25
Clearance Ratio ($\overline{c}$)	0.001
No. of Recesses for Symmetric Bearing Configuration	12
No.of rows	2
Offset Factor (δ)	0.75, 1.0, 1.25
Wear Depth Parameter ($\overline{\delta}_w$)	0.0,0.3,0.5
Operating Parameters	
Speed Parameter (Ω)	1.0
Eccentricity Ratio (ε)	0.1- 0.9
Concentric Design Pressure Ratio (β^*)	0.5

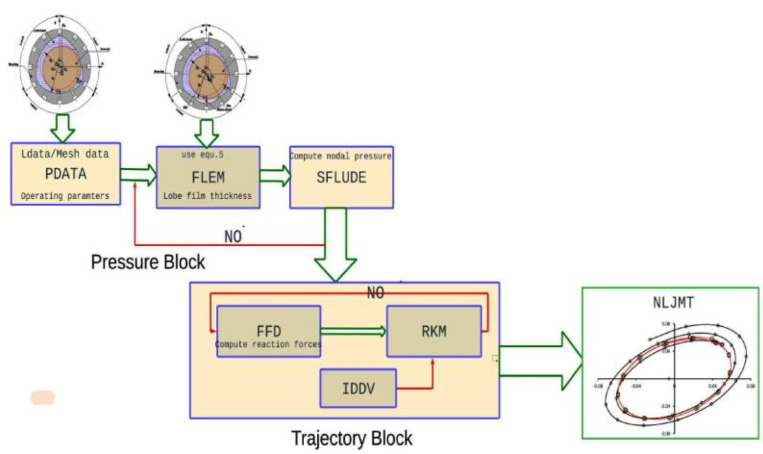


Figure 2: The computation result using the solution procedure presented in the overall self-explanatory iterative procedure.

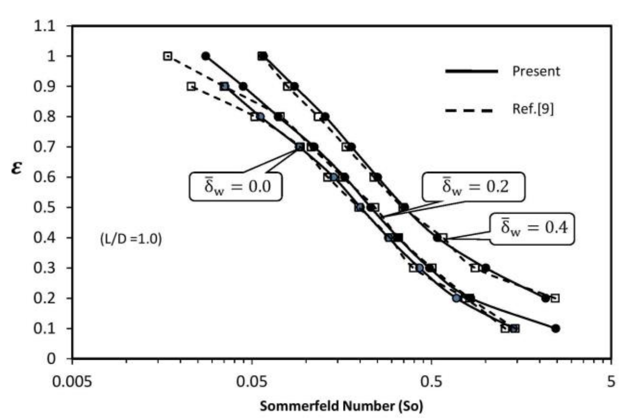


Figure 3: Worn out bearing result for hydrodynamic journal bearing.

The obtained results exhibit well consistency with the results of Hashimoto et al., (1986). The resulting journal trajectories have been compared with the findings from the momentary journal response with those from Sinhasan and Goyal, (1992), who had previously published their findings. The comparison shows that the nonlinear equations anticipate unstable motion nonetheless, a limited cycle is what the linear equations forecast when $\overline{M}_j$=$\overline{M}_c$. These trajectories for the hydrodynamic journal bearing exhibit a strong agreement as shown in Figure 4. The motion trajectories of the 3-lobe bearing journal center have been computed and presented (Figure: 5-8) using nonlinear analysis when $\overline{M}_j$=0.9$\overline{M}_c$, $\overline{M}_j$=$\overline{M}_c$ and $\overline{M}_j$=1.1$\overline{M}_c$ for the offset factors (δ=0.75,1.0,1.25), and wear depth parameter ($\overline{\delta}w = 0.0,0.3,0.5$).

The plot for trajectories when $\overline{M}_j$=0.9$\overline{M}_c$ has been obtained for the operation of the unworn ($\overline{\delta}_w = 0.0$) and worn out ($\overline{\delta}_w = 0.3,0.5$) bearing with under an external load of $\overline{W}_0 = 1.5$ and has been shown in Figure 5 (a)-(f). When $\overline{M}_j$=0.9$\overline{M}_c$, a stable cycle is predicted by the nonlinear equation of motion. When the requirements for stability are met the system becomes asymptotically stable. Figure 5 (a), (b) and (c) depicts that when 3-lobe and circular journal bearing $\delta = 1.0$ operates with at $\overline{W}_0 = 1.5$, A steady motion is predicted by a nonlinear equation of motion for $\delta = 0.75, \delta = 1.0$ and $\delta = 1.25$ when $\overline{M}_j$=0.9$\overline{M}_c$. As depicted in Figure 5(c), 3-lobe journal bearing at $\delta = 1.25$ and wear depth parameter ($\overline{\delta}_w = 0.5$) maintains the fluid layer thickness below the circular journal bearing's ($\delta = 1.0$) minimal requirement throughout the entire load range. Furthermore, under the identical operating circumstances, the 3-lobe journal bearing traces a bigger orbit when the bearing non circularity parameter i,e. offset is less than one ($\delta = 0.75$). Further it may be seen from the Figure 5 (d)-(f) that an increase in radial clearance due to wear ($\overline{\delta}_w = 0.5$) results in greater eccentricity during the bearing's operation. It improves the bearing stability due to adequate availability of lubricant in the worn zone. Figure 6(a)–(f) illustrates the transient journal orbits for different noncircular bearing geometries when journal mass reaches critical mass. Figure 6(a)–(c) shows the non-linear trajectories for $\overline{M}_j$=$\overline{M}_c$ when worn and unworn 3- lobe symmetric non-recessed hybrid journal bearing operates at $\overline{W}_0 = 1.5$. Figure 6(a) and 6(b) depicts limit cycle motion for circular and non-circular unworn ($\overline{\delta}_w = 0.0$) and worn ($\overline{\delta}_w = 0.5$) journal bearings. In limit cycle condition, the journal's locus within the bearing neither approaches a fixed point nor diverges infinitely. Instead, it forms a cyclical pattern. The system's stability is delicately balanced, and even a slight change in lobe geometry and worn-out zone or both, has the potential to shift the bearing system from stability to instability or vice versa. This particular trajectory of the journal represents the critical threshold state of the bearing at that precise moment. It is evident from the Figure 6(c) that an increase in radial clearance caused by wear ($\overline{\delta}_w = 0.5$) can lead to an improvement in the bearing's stability, consequently the journal anticipates stable motion for various bearing geometries (δ =0.75, 1.0 and 1.25) as a result of this enhanced stability. From Figure 6 (d)-(f), it may be noticed that an increase in the offset factor (δ =1.25) leads to a greater bearing clearance in comparison to a circular bearing (δ =1.0). This, in turn, concentrates the pressure distribution within a narrower region. Consequently, the tangential component of force is effectively counteracted by the applied load. Consequently, when the non-circular bearing offset factor exceeds one, 3-lobe bearing exhibits stable motion during operation. When $\overline{M}_j$=$\overline{M}_c$, journal motion path follows stable motion patterns for all bearing configurations (δ =0.75, 1.25) including circular journal bearing (δ =1.0). With reference to Figure 6(d)-(f), for unworn non-recessed hybrid journal bearing ($\overline{\delta}_w = 0.0$) the value of the minimum fluid film thickness is more, and the journal traces a bigger orbit shifting towards the left. Figure 6 (d)-(f) also depicts that modifying the bearing geometry (δ=1.25) contributes to maximum pressure ($\overline{P}_{max}$) and effectively stabilizes the journal motion. As the radial clearance caused by wear damage seen to grow ($\bar{\delta}_w = 0.3,0.5$), all bearing configurations (δ =0.75, 1.0 and1.25) provides non-linear trajectory stable motion and operates at lower values of $\overline{h}_{min}$.

Figure 7 (a)-(f) depicts the journal centre motion path of a 3-lobe non-recessed journal bearing under a specific external load condition ($\overline{W}_0 = 1.5$). when $\overline{M}_j$=1.1$\overline{M}_c$. Figure7 (a) indicates the unstable journal motion trajectories for unworn bearing for the values of $\delta = 0.75,1.0, 1.25$.It is obvious that these equation of motion foresees a wobbly movement if the mass of the journal surpasses the critical threshold ($\overline{M}_j$=1.1$\overline{M}_c$).This happens as the journal rotates within the bearing, the fluid pressure within the bearing varies, causing the journal to experience dynamic motion. Because of this journal trajectory may not follow a simple path due to the complex interaction between the fluid film and the rotating journal. The same trend is also depicted in Figure 7(b) &(c) for damaged bearing surfaces ($\bar{\delta}w = 0.3,0.5$) for all bearing configurations ($\delta = 0.75,1.0, 1.25$). As anticipated, Figure 7(d)-(f) shows unstable nonlinear motion trajectories for different wear depth parameters ($\bar{\delta}w = 0.0, 0.3,0.5$), the system exhibits instability when $\overline{M}_j$=1.1$\overline{M}_c$. Thus, it becomes crucial to exercise greater vigilance and attention. Moreover, it is noticeable from Figure 7(d) and 7(e) that with an increase in the$\overline{M}_j$, the size of the journal center correspondingly also expands and bearing journal traces large orbit. When, $\overline{M}_j$=1.1$\overline{M}_c$ the combined effects of the wear depth parameter ($\bar{\delta}w$) and bearing geometry (δ) significantly impact the size and form of trajectories to fit inside the bearing's smallest clearance. Figure 8 depicts the variation of $\bar{\omega}_{th}$ for different values of an offset factor (δ=1.25, 1.0) and wear depth parameter ($\bar{\delta}w = 0.0,0.3,0.5$). The observed stability threshold speed margin is more for 3-lobe unworn non-recessed journal bearing operates at δ=1.25. However, it may also be noticed that for the increased radial clearance more than 30% also provides more stability of $\bar{\omega}_{th}$. For the sake of brevity, variation of $\bar{\omega}_{th}$ is presented only for circular and 3-lobe (δ=1.25) hybrid journal bearings.

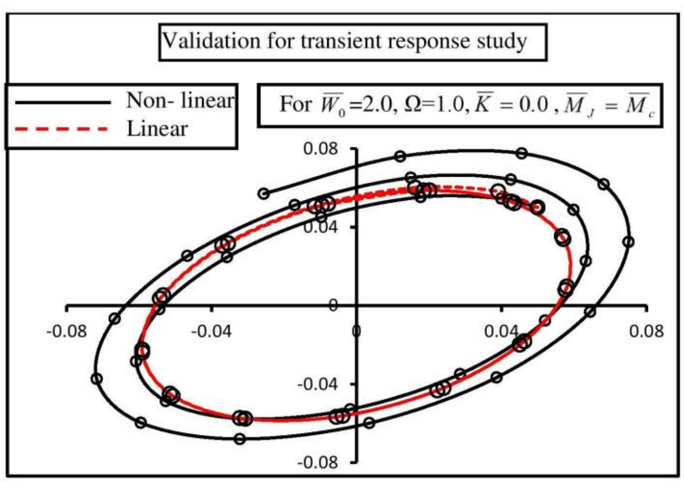


Figure 4: Trajectories for the hydrodynamic journal bearing.

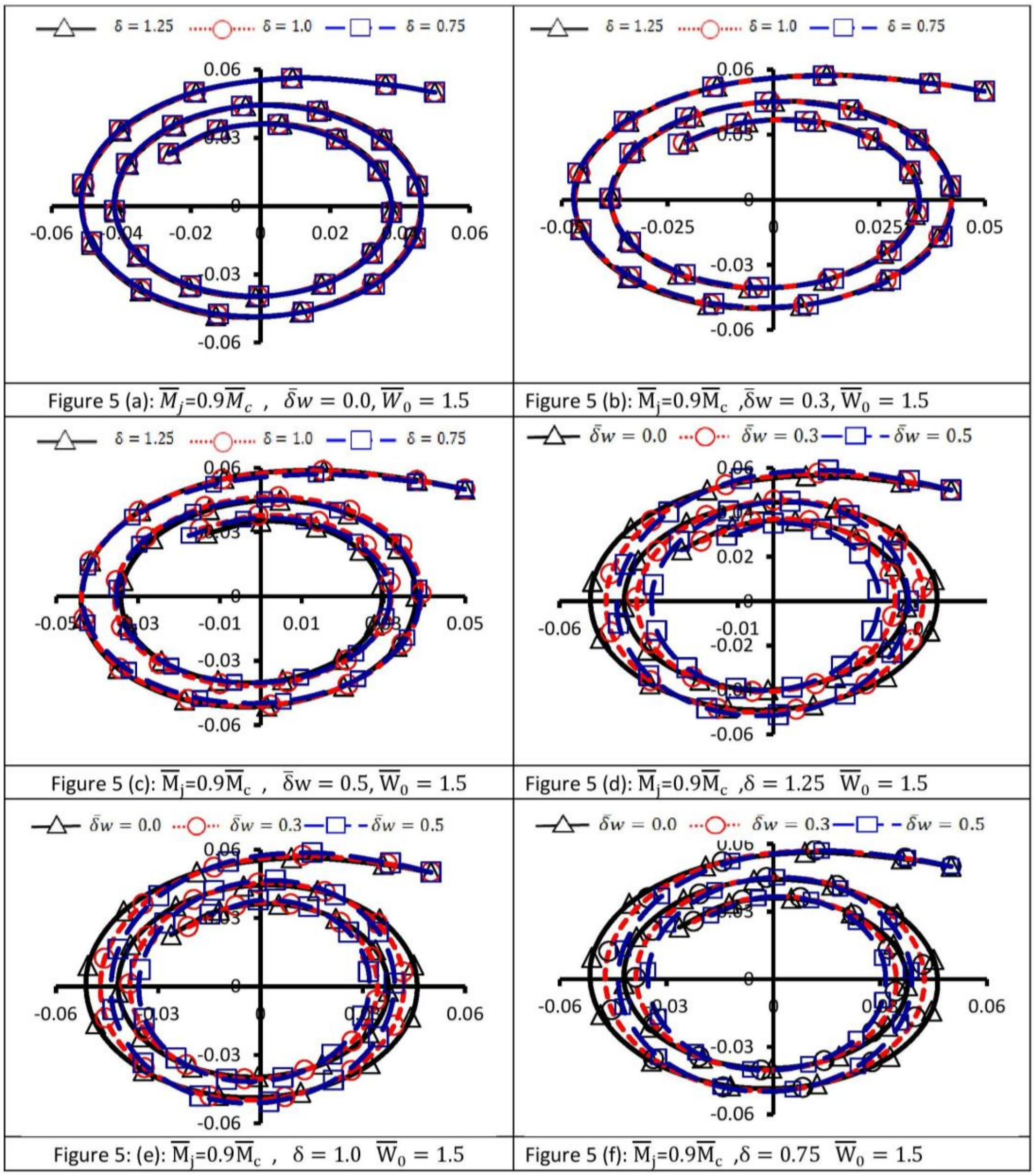


Figure 5 (a): $\overline{M}_j$=0.9$\overline{M}_c$, $\bar{\delta}w = 0.0$, $\overline{W}_0 = 1.5$

Figure 5 (b): $\overline{M}_j$=0.9$\overline{M}_c$,$\bar{\delta}w = 0.3$, $\overline{W}_0 = 1.5$

Figure 5 (c): $\overline{M}_j$=0.9$\overline{M}_c$, $\bar{\delta}w = 0.5$, $\overline{W}_0 = 1.5$

Figure 5 (d): $\overline{M}_j$=0.9$\overline{M}_c$,$\delta = 1.25$ $\overline{W}_0 = 1.5$

Figure 5: (e): $\overline{M}_j$=0.9$\overline{M}_c$, $\delta = 1.0$ $\overline{W}_0 = 1.5$

Figure 5 (f): $\overline{M}_j$=0.9$\overline{M}_c$,$\delta = 0.75$ $\overline{W}_0 = 1.5$

Figure 5: Motion trajectories of the 3-lobe bearing journal center.

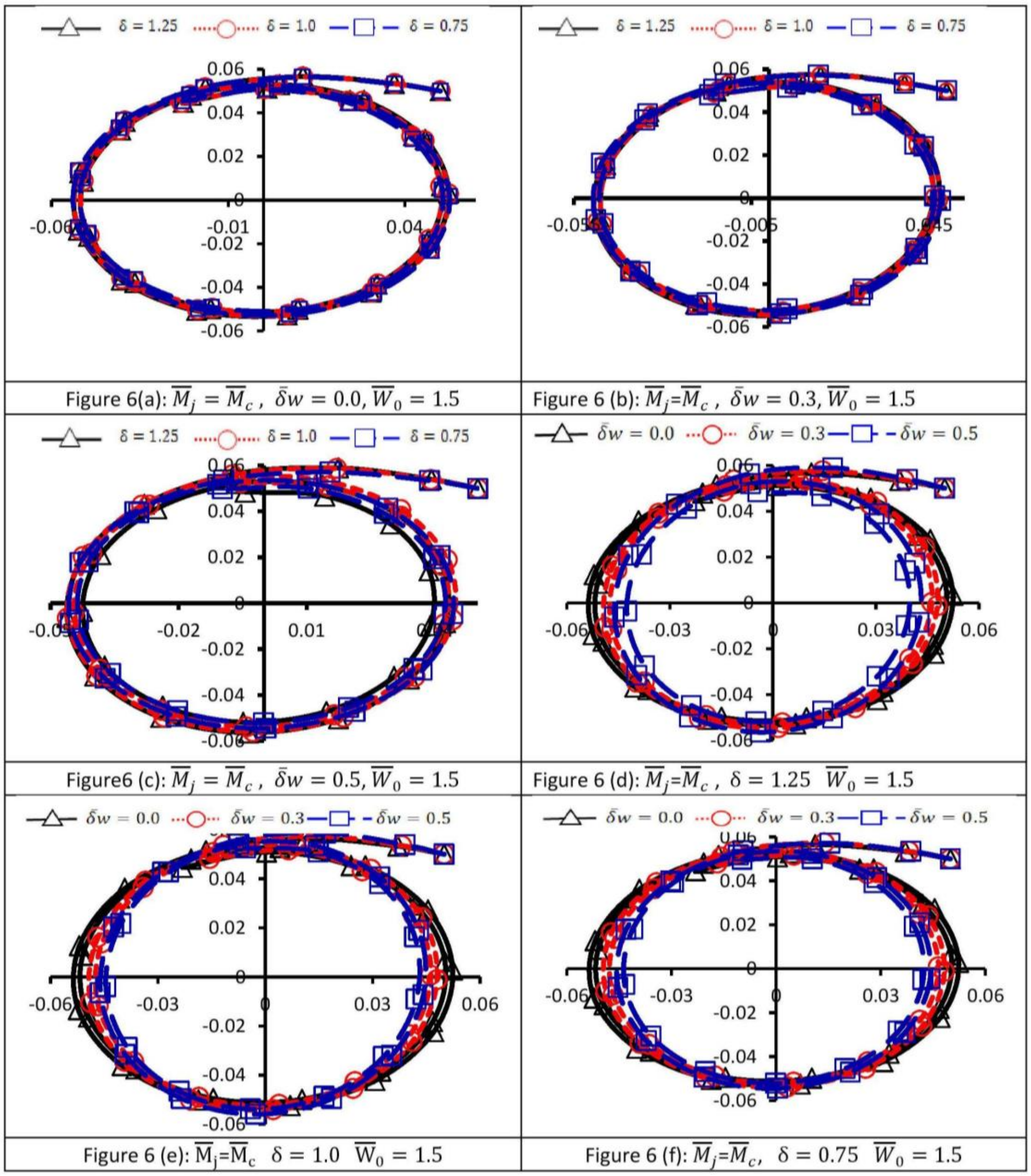


Figure 6(a): $\overline{M}_j = \overline{M}_c$, $\bar{\delta}w = 0.0, \overline{W}_0 = 1.5$

Figure 6 (b): $\overline{M}_j$=$\overline{M}_c$, $\bar{\delta}w = 0.3, \overline{W}_0 = 1.5$

Figure6 (c): $\overline{M}_j = \overline{M}_c$, $\bar{\delta}w = 0.5, \overline{W}_0 = 1.5$

Figure 6 (d): $\overline{M}_j$=$\overline{M}_c$, $\delta = 1.25$ $\overline{W}_0 = 1.5$

Figure 6 (e): $\overline{M}_j$=$\overline{M}_c$ $\delta = 1.0$ $\overline{W}_0 = 1.5$

Figure 6 (f): $\overline{M}_j$=$\overline{M}_c$, $\delta = 0.75$ $\overline{W}_0 = 1.5$

Figure 6: limit cycle motion for circular and non-circular unworn and worn journal bearings.

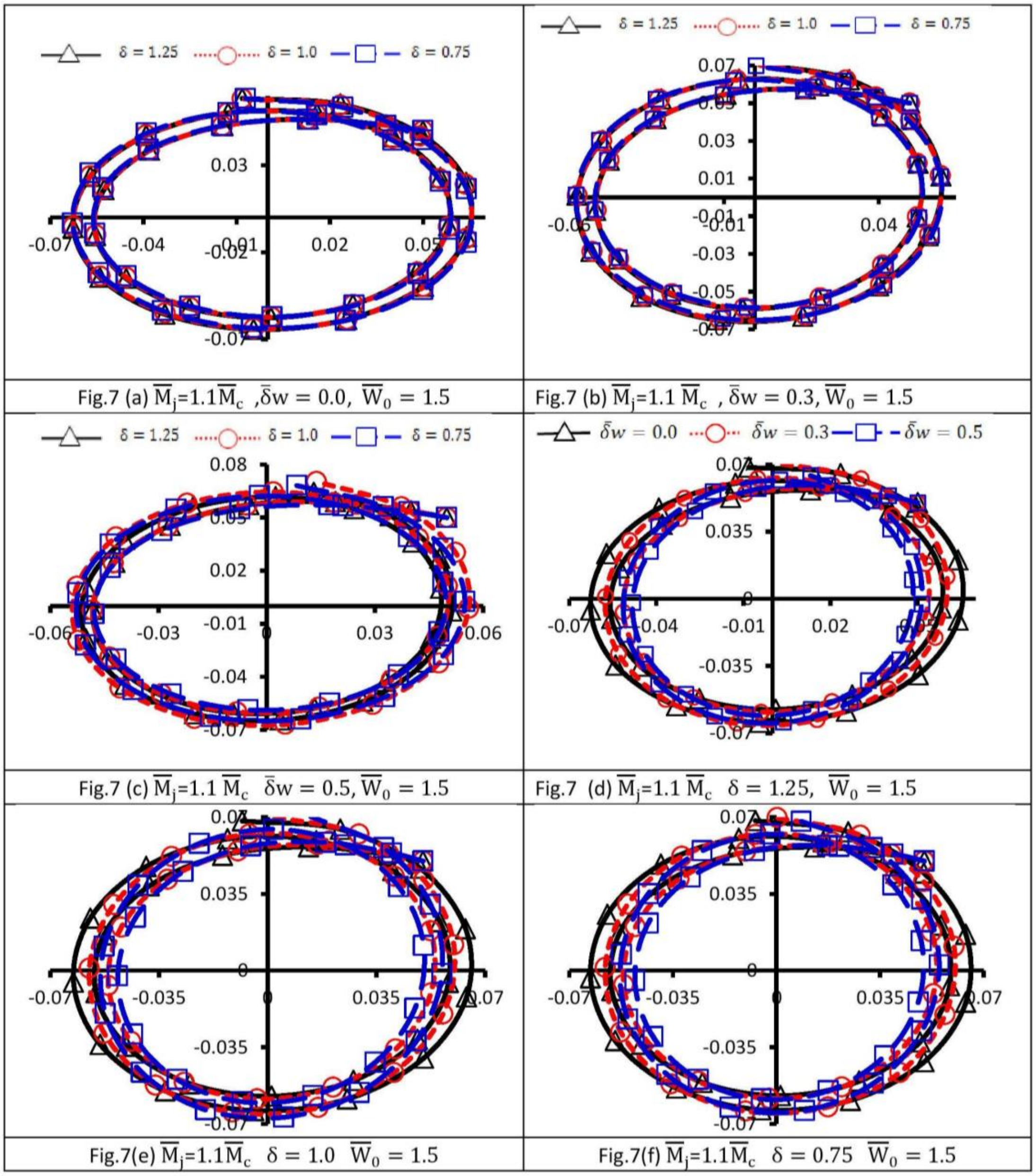


Figure 7: Unstable nonlinear motion trajectories for different wear depth parameters.

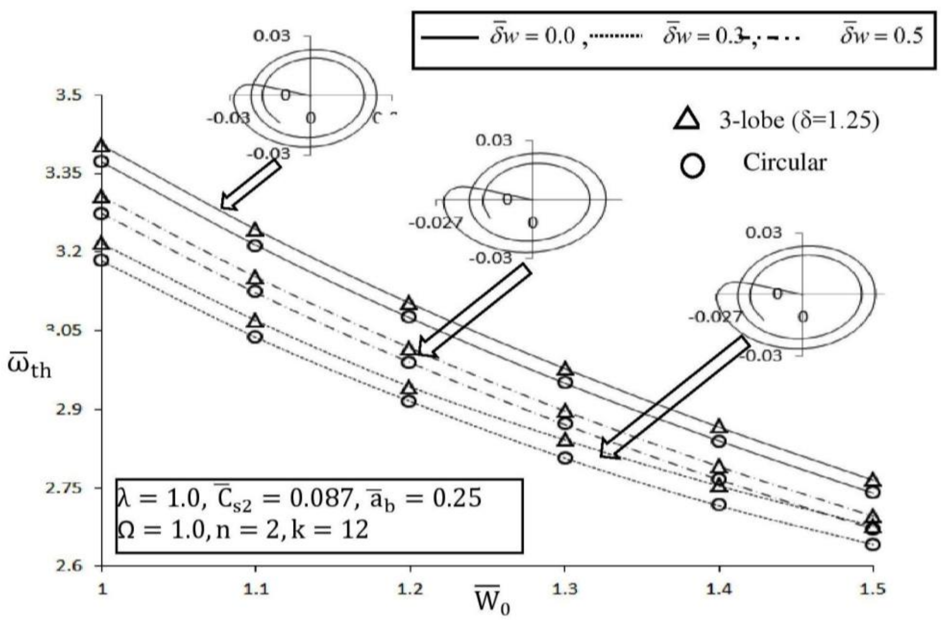


Figure 8: variation of $\bar{\omega}_{th}$ for different values of an offset factor.

CONCLUSIONS

The current numerical study examined the influence of wear on the stability of a 3-lobe capillary compensated symmetric non-recessed journal bearing. The study focused on analyzing trajectories of the journal orbit motion to assess the bearing system's stability. Through extensive numerical simulations, following significant findings are drawn from the study:

The combined effects of non-circular bearing geometries (δ) and the wear depth parameter ($\bar{\delta}$w) have a significant impact on the transient journal center motion of 3-lobe non-recessed journal bearing. Specifically, when δ>1 and the wear depth parameter ($\bar{\delta}$w) exceeds 0.3, the nonlinear motion analysis indicates a remarkable enhancement in the safety margin concerning stability. Properly selecting the bearing geometry or offset factor can mitigate the influence of the wear depth parameter ($\bar{\delta}$w) on the stability of the journal bearing system.

NOMENCLATURE

$C_1 = c$	:Clearance due to circumscribed circle on the bearing, mm
C_2	:Clearance due to inscribed circle on the bearing, mm
C_{ij}	:Damping coefficient (i,j=1,2) N.s
D	:Diameter of the journal, mm
F	:Fluid-film reaction ($\frac{\partial h}{\partial t} \neq 0$), N
F_o	:Fluid-film reaction ($\frac{\partial h}{\partial t} = 0$), N
L	:Length of the journal bearing, mm
M_c	:Critical mass of journal, Kg
M_J	:Journal mass, Kg

N	:Rotational speed, rps
O_b	:Bearing center
O_J	:Journal center
Q	:Bearing lubricant flow, $mm^3\ s^{-1}$
Q_R	:Flow through restrictor, $mm^3\ s^{-1}$
R_J	:Radius of journal, mm
R_L, R_b	:Radius of lobe and bearing
S_{ij}	:Fluid film stiffness coefficient (i,j=1,2)(N mm^{-1})
U	:Surface speed $((= \omega_J \times R_J)$, mm s^{-1}
W_0	:External load, N
X,Y,Z	:Cartesian coordinates
X_J, Z_J	:Coordinates of steady-state equilibrium journal center from geometric center of bearing, mm
a_b	:Axial bearing land width, mm
c	:Radial clearance, mm
e	:Journal eccentricity, mm
h	:Total fluid film thickness, mm
h_0	:Fluid film thickness without wear, mm
h_{min}	:Minimum fluid film thickness, mm
n	:No.of rows
p	:Pressure, Nmm^{-2}
P_c	:Hole pressure $(\partial h/\partial t \neq 0)$, Nmm^{-2}
P_s	:Supply pressure, Nmm^{-2}
t	:Time, s

Non-dimensional parameters

$\overline{C}_{ij}$	$= C_{ij}(c^3 / \mu R_J^4)$, Damping coefficients
$\overline{C}_{s2}$	$= \left[\frac{3\pi a^4}{12c^3 l_c}\right]$, Restrictor design parameter for capillary restrictor
$\overline{F}$	$=(F/p_s R_J^2)$
$\overline{F}_o$	$=(F_o/p_s R_J^2)$
$\overline{M}_c, \overline{M}_J$	$= (M_c, M_J)\left(\frac{c^2 p_s}{\mu R_J^2 \omega_J}\right)$
$\overline{S}_{ij}$	$=S_{ij}(c / p_s R_J^2)$
$\overline{W}_0$	$= W_o/p_s R_J^2$, External load
$\overline{X}_J$	$=X_J/c$
$\overline{Z}_J$	$=Z_J/c$
$\bar{X}_L^i, \bar{Z}_L^i$	$=(X_L^i, Z_L^i)/c$
$\bar{a}_b$	$= a_b/L$, Land width ratio
$\bar{h}_0$	$=h_0/c$
$\bar{p}, \bar{p}_{max}$	$=(p, p_{max})/p_s$
$\bar{t}$	$=t(c^2 p_s/\mu R_J^2)$
$\bar{\delta}_w$	=Wear depth, δ_w/c
Ω	$= \omega_J(\mu R_J^2/c^2 p_s)$, Speed parameter

α, β	$= (X, Y)/R_J$, Circumferential coordinates
β^*	$=p/p_s$, concentric design pressure ratio $(\partial h/\partial t = 0)$,
δ	$=C_1/C_2$
$\partial\bar{h}$	Fluid film thickness for worn segment
$\bar{\omega}_{th}, \bar{\omega}_d$	$= (\omega_{th}, \omega_d)/\omega_J$

Greek Symbols

α_b, α_e	:Angles corresponding to beginning and end of the worn surface
λ	:Aspect ratio, L/D
ε	:Eccentricity ratio, e/c
δ	:Offset factor, C_1/C_2
ω_J	:Journal rotational speed, rad s^{-1}
ω_{th}	:Threshold speed, rad s^{-1}

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